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К ОЦЕНКЕ ОБЛАСТИ ПРИТЯЖЕНИЯ ПОЛОЖЕНИЯ РАВНОВЕСИЯ НЕЛИНЕЙНЫХ СИСТЕМ УПРАВЛЕНИЯ

Синтез нелинейных систем управления, по-прежнему, является сложной задачей, поэтому многие исследователи пытаются найти эффективные способы и методы решения этой проблемы. В результате таких исследований было разработано несколько методов синтеза систем управления для нелинейных объектов, каждый из которых даёт системы с различными свойствами. Поэтому возникла необходимость сравнить некоторые методы, чтобы определить, какой из них является достаточно простым и позволяет найти нелинейную систему с лучшими свойствами. С этой целью, в данной работе сравниваются допустимые области начальных условий, при которых созданные различными методами нелинейные системы управления являются работоспособными. Рассматриваются два аналитических метода проектирования систем управления различными нелинейными техническими объектами, такими как мобильные роботы и многие другие объекты. Это алгебраический полиномиально-матричный метод, использующий квазилинейную модель, и метод линеаризации обратной связью, использующий приведение заданных нелинейных уравнений объекта к форме Бруновского. Оба рассмотренных метода дают ограниченную область притяжения положения равновесия полученных систем управления, поэтому эти системы могут работать только с ограниченными начальными условиями. В статье приведен численный пример проектирования систем управления для одного объекта этими двумя методами. Оценки областей притяжения равновесия этих систем определяются с помощью MATLAB. В результате установлено, что алгебраической полиномиально-матричной метод позволяет обеспечить большую область допустимых начальных условий, по сравнению с методом линеаризации обратной связью. Кроме того, алгоритм синтеза нелинейных систем управления алгебраическим полиномиально-матричным методом является более простым и полностью выполняется на компьютере. Это позволяет считать, что решение задач проектирования систем управления нелинейными объектами целесообразнее выполнять алгебраическим полиномиально-матричным методом.

Нелинейный объект; ограниченные начальные условия; область притяжения; алгебраический полиномиально-матричный метод; метод линеаризующих обратных связей.

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TO ESTIMATION OF ATTRACTION AREA OF EQUILIBRIUM OF NONLINEAR CONTROL SYSTEMS

Designing nonlinear control systems is still difficult so many researchers are trying to find some useful ways and methods to solve this problem. As a result of such research, some methods have been seen trying to design a good enough control system for nonlinear plants. But a disadvantage of these methods is the complexity, so it created a need to compare some methods to determine which one is the easiest method to design a control system for nonlinear plants. It was found a way to compare two methods, which is comparing the regions of initial conditions of the systems which are designed using these methods. Two analytical nonlinear control systems design methods are compared on the example of the design control systems mobile robots. The algebraic polynomial-matrix method uses a quasilinear model, and the feedback linearization method uses particular feedback. Both considered methods give a bounded domain of equilibrium attraction, therefore the obtained control systems can be operated only with bounded initial conditions. The numerical example of designing the control systems for one object by these methods and the estimates of the attraction areas of the system's equilibriums of these systems are given in the paper. As a result of this paper, it was found that using the algebraic polynomial-matrix method will get a bigger cross section of initial conditions of the plant's variable than the same cross section which is given by the feedback linearization method.

Nonlinear plant; bounded initial condition; attraction area; algebraic polynomial-matrix method; linearizing feedbacks method.

Introduction. Real-world systems are inherently nonlinear in nature at least when considered over a wide operating range [1]. Recently, an active interest in the design and analysis of nonlinear control systems has been shown in much research like process control, biomedical engineering, robotics, and spacecraft control [2]. One of the most effective reasons behind the growing interest in nonlinear control includes the need to deal with model uncertainties and design simplicity [3].

A lot of researchers have tried designing an effective nonlinear control. New research has been conducted to simplify the process of designing nonlinear systems using transformation methods [4, 5]. Usually, the nonlinear plant equations are transformed into such forms as the feedback linearization method [4–6], regularization method [7], passivity method [8–10], backstepping method [11], Jordan controlled form method [12], quasilinear model method [13, 14], position control method [15, 16], and others.

The aim of this paper is to find the areas of the initial conditions in which the control systems for one nonlinear plant are still asymptotically stable. In addition to finding a control system that is designed by two analytical methods and making a comparison between them to decide which design method gives a wider area of attraction under initial conditions.

This paper consists of 5 parts. The first is an introduction, the second one is a definition of the algebraic polynomial-matrix (APM) method, which uses the quasilinear model, the third part is a short definition of the feedback linearization method, and the fourth part is devoted to giving an example in order to evaluate the resulted area of attraction which is given by these two methods, and the last part is the conclusion.

Algebraic polynomial-matrix method. This method is used for the design a control system of linear or nonlinear objects of arbitrary order, which are given as quasilinear model and have the form

$$\dot{x} = A(x)x + b(x)\,u\,,\ \ y = c^T(x)x \tag{1}$$

In this case, the desired control model is given in the form

$$u = u(g,x) = k_0(x)g - k^T(x)x = k_0(x)g - [k_1(x)x_1 + k_2(x)x_2 + \ldots + k_n(x)x_n], \tag{2}$$

where $k_i(x)$ are the coefficients of the matrix-row $k^T(x)$ calculated during the design, which are the feedback coefficients on state variables x_i in a closed system, $i = \overline{1,\, n}$; and $g = g(t)$ is the setting action. Usually, $g(t)\ = g_0 * 1(t)$ [17, 18].

Substituting (2) into (1) shows that the mathematical model of a closed system will also have the form corresponding to the structure of a quasilinear model, i.e.

$$\dot{x} = D(x)x + k_0(x)b(x)g\,, \tag{3}$$

where $D(x)$ is also a functional matrix of the same dimension as $A(x)$. The structure of the matrix $D(x)$, as follows from the substitution, is determined by the following expression:

$$D(x) = A(x) - b(x)\cdot k^T(x)\cdot \tag{4}$$

For determining $k_i(x)$ are found polynomials which have the form

$$A(p,x) = \det(pE - A(x)) = p^n + \alpha_{n-1}(x)p^{n-1} + \ldots + \alpha_1(x)p + \alpha_0(x), \tag{5}$$

$$V_i(p,x) = e_i\cdot \mathrm{Adj}\big(pE - A(x)\big)\cdot b(x) = v_{i,n-1}p^{n-1}(x) + \ldots + v_{i,1}(x)p + v_{i,0}(x), \tag{6}$$

In these expressions $\alpha_i(x)$ are the coefficients of the characteristic polynomial of the matrix $A(x)$ of the controlled object (1); e_i - i-th row of the identity $n\times n$-matrix E;

$\mathrm{Adj}\big(pE - A(x)\big)$ is the attached matrix; $v_{ij}(x)$ are the coefficients of the polynomials $V_i(p,x)$, $i=\overline{1,\,n}$, $j=\overline{0,\,n-1}$.

A polynomial is formed

$$D^*(p)=\prod_{i=1}^{n}(p-p_i^*)=p^n+\delta_{n-1}p^{n-1}+\delta_{n-2}p^{n-2}+....+\delta_1 p+\delta_0 \cdot \quad (7)$$

where δ_i are any positive numbers for which the roots of the polynomial $D^*(p)$ are real, distinct and negative. Next, the difference of polynomials $D^*(p)$ and $A(p,x)$ and its coefficients are determined:

$$R(p,x)=D^*(p)-A(p,x)=\bar{\delta}_{n-1}p^{n-1}+\bar{\delta}_{n-2}(x)p^{n-2}+....+\bar{\delta}_1(x)p+\bar{\delta}_0(x)\cdot \quad (8)$$

A system of algebraic equations is compiled:

$$\begin{bmatrix} v_{1,0} & v_{2,0} & \cdots & v_{n,0} \\ v_{1,1} & v_{2,1} & \cdots & v_{n,1} \\ \vdots & \vdots & \vdots & \vdots \\ v_{1,n-1} & v_{2,n-1} & \cdots & v_{n,n-1} \end{bmatrix}\cdot\begin{bmatrix} k_1 \\ k_2 \\ \vdots \\ k_n \end{bmatrix}=\begin{bmatrix} \bar{\delta}_0 \\ \bar{\delta}_1 \\ \vdots \\ \bar{\delta}_{n-1} \end{bmatrix}. \quad (9)$$

The matrix $D(x)$ is calculated by use expression (4). The coefficient $k_0(x)$ from (2) is found with condition $y^o = g_0$ by the formula

$$k_0(x^\circ)=\frac{-1}{c^T(x^\circ)D^{-1}(x^\circ)b(x^\circ)}, \quad (10)$$

where $x^\circ=\lim x(t)_{t\to\infty}$.

Feedback Linearization method. To apply this method, the equation (1) is represented:

$$\dot{x}=\tilde{A}(x)+b(x)u_2, \quad (11)$$

A controllability matrix has the form in this case

$$U_2=\left(b,[\tilde{A},b],....,[\tilde{A},ad_{\tilde{A}}^{n-1}b]\right), \quad (12)$$

in expression (12) $ad_{\tilde{A}}^{0}b=b$; $ad_{\tilde{A}}^{1}b=[\tilde{A},\ b]=(\partial b/\partial x)\tilde{A}-(\partial\tilde{A}/\partial x)b$;

$ad_{\tilde{A}}^{i}b=[\tilde{A},ad_{\tilde{A}}^{i-1}b]$, is a derivative of vector field b in direction of $\tilde{A}(x)$ [21]. If the determinate of controllability matrix (12) doesn't equal to zero and columns are involute, the transformation $z(x) = T_1(x)$ are determined from conditions

$$\frac{\partial T_1}{\partial x}ad_{\tilde{A}}^{i}b=0,\quad i=\overline{0,\,n-2},\ \frac{\partial T_1}{\partial x}ad_{\tilde{A}}^{n-1}b\neq 0\cdot \quad (13)$$

The transformation $z(x)$ allows to find feedback $u_2(x)$, which converts the system (11) into linear Brunovsky form with control v. Then the linear stabilizing control $v = v(z)$ can be determined very easily, and the transformation $z(x)$ will give required control $u_2 = u_2(x) = v(z(x))$.

To evaluate the area of attraction of the equilibrium of nonlinear control systems, we need firstly to design the control, which makes the system asymptotically system. Then max and min of each of the initial conditions of the plant's variables in which the system is still stable determined, to do that, will take an example.

Example. Suppose a pendulum is described as a form [19]

$$A(x)=\begin{bmatrix}0 & 1 & 0\\ 5w(x_1) & 0 & 2\\ 7w(x_1) & 0 & 1\end{bmatrix},\ b(x)=\begin{bmatrix}0\\2\\1\end{bmatrix}, \tag{14}$$

where $w(x_1)=x_1^{-1}\sin x_1$; $A(x)=[a_{ij}(x)]$ is a functional $n\times n$-matrix, $b(x)=[b_i(x)]$ is functional *n*-vector [21]. To find the stabilizing control for (14) by the considered methods.

Design by using algebraic polynomial-matrix method. According to this method firstly we need to check satisfying of the controllability condition [13]. In this case $\det U_1(x)=-36x_1^{-1}\sin x_1$. So we can find a control system only if $|x_1|<\pi$. The polynomials can be determined as [20]:

$$A(p,x)=\det(pE-A(x))=p^3-p^2+\alpha_1(x)p+\alpha_0(x),$$
$$V_1(p,x)=e_1\text{Adj}(pE-A(x))b=2p,$$
$$V_2(p,x)=e_2\text{Adj}(pE-A(x))b=2p^2 \tag{15}$$
$$V_3(p,x)=e_3\text{Adj}(pE-A(x))b=p^2+9w(x_1),$$

where $\alpha_1(x)=-5w(x)$; $\alpha_0(x)=-9w(x)$, e_i is *i*-th a line of a unit matrix of *E*.

Let the desirable Hurwitz polynomial $D^*(p)=p^3+\delta_2^*p^2+\delta_1^*p^1+\delta_0^*$ of the matrix $D_1(x)=A(x)-b(x)k^T(x)$ and $R(p,x)=D^*(p)-A(p,x)=(\delta_2^*+1)p^2+(\delta_1^*+5w(x_1)p+\delta_0^*+9w(x_1)$. Therefore, polynomials (15) will led to create the algebraic system:

$$\begin{bmatrix}0 & 0 & 9w(x_1)\\ 2 & 0 & 0\\ 0 & 2 & 1\end{bmatrix}\cdot\begin{bmatrix}k_1(x)\\k_2(x)\\k_3(x)\end{bmatrix}=\begin{bmatrix}\delta_0^*+9w(x_1)\\ \delta_1^*+5w(x_1)\\ \delta_2^*+1\end{bmatrix},\ |x_1|<\pi, \tag{16}$$

where $k_i(x)$ are some nonlinear functions, *i* = 1, 2, … *n* [21]. The solution of system (16) gives the required control:

$$u_1(x)=-[0{,}5\delta_1^*+2{,}5w(x_1)]x_1-[0{,}5\delta_2^*-\delta_0^*/18w(x_1)]x_2-\ -[1+\delta_0^*/9w(x_1)]x_3,\ |x_1|<\pi.$$

Design by using feedback linearization method. According to this method, here $\tilde{A}(x)=\begin{bmatrix}x_2 & 5\sin x_1+2x_3 & 7\sin x_1+x_3\end{bmatrix}^T$, $b(x)=\begin{bmatrix}0 & 2 & 1\end{bmatrix}^T$. To find the transformation $z(x)$ we need first to find the controllability matrix $U_2=\left(b\ ad_{\tilde{A}}^1b\ ad_{\tilde{A}}^2b\right)=\left(b\ [\tilde{A},b]\ \left[\tilde{A},[\tilde{A},b]\right]\right)$. In this case

$$ad_{\tilde{A}}b=(\partial b/\partial x)\tilde{A}-(\partial\tilde{A}/\partial x)\cdot b=[-2\quad -2\quad -1]^T. \tag{17}$$

$$ad_{\tilde{A}}^2b=ad_{\tilde{A}}(ad_{\tilde{A}}\,\text{b})=(\partial(ad_{\tilde{A}}b)/\partial x)\tilde{A}(x)-(\partial\tilde{A}/\partial x)ad_{\tilde{A}}b= \tag{18}$$
$$=[2\quad 10\cos x_1+2\quad 14\cos x_1+1]^T.$$

According to (17), (18), the controllability matrix has form

$$U_2(x)=\begin{bmatrix}0 & -2 & 2\\ 2 & -2 & 10\cos x_1+2\\ 1 & -1 & 14\cos x_1+1\end{bmatrix}. \qquad (19)$$

The $\det U_2 = 36\cos x_1$ and columns of the matrix (19) form an involute set, i.e. the controllability condition is satisfied if only $|x_1| < \pi/2$ [20].

Now we can define the transformation $z(x)=T(x)=[T_1(x)\ T_2(x)\ T_3(x)]^T$ starting from the function $T_1(x)$; this function is determined in [20]:

$$T_{1x}(x)\, b = 2\,T_{1,x2} + T_{1,x3} = 0, \qquad (20)$$

$$T_{1x}(x) ad_{\tilde{A}} b = -2\,T_{1,x1} - 2\,T_{1,x2} - 2\,T_{1,x3} = 0, \qquad (21)$$

$$T_{1x}(x) ad_{\tilde{A}}^2 b = -2T_{1,x1} + (10\cos x_1 + 2)T_{1,x2} + (14\cos x_1 + 1)T_{1,x3} \neq 0. \qquad (22)$$

The conditions (20), (21) and (22) are used to define the function $T_1(x)=T_1(x_1,x_2,x_3)$ which can be any function that satisfies these three conditions [20]. From the conditions (20) we can say that $T_1(x)$ depends only on x_2 and x_3. On the basis of a condition (21), (22) it is possible to accept $T_1(x) = x_2 - 2x_3 = z_1$ as the simplest function.

The function $T_{2x}(x)$ is defined from expression $T_2(x)=T_{1x}\tilde{A}(x) = -9\sin x_1 = z_2$. Similarly, $T_{3x}(x)$ can be defined by expression: $T_3(x) = T_{2x}\tilde{A}(x) = -9x_2\cos x_1 = z_3$. It will easily be convinced that the transformation

$$z(x)=T(x)=\begin{bmatrix}x_2-2x_3\\ -9\sin x_1\\ -9x_2\cos x_1\end{bmatrix} \qquad (23)$$

transforms the equation (11) where $\tilde{A}(x)=\begin{bmatrix}x_2 & 5\sin x_1+2x_3 & 7\sin x_1+x_3\end{bmatrix}^T$, $b(x)=\begin{bmatrix}0 & 2 & 1\end{bmatrix}^T$ into the linear equation of Brunovsky form:

$$\dot{z}=\begin{bmatrix}z_2 & z_3 & v\end{bmatrix}^T. \qquad (24)$$

Control v(z) at which the linear system (24) is asymptotically stabile evidently has an appearance $v=-\delta_0 z_1 - \delta_1 z_2 - \delta_2 z_3$, at which the equation (24) passes into the equation

$$\dot{z}=\begin{bmatrix}0 & 1 & 0\\ 0 & 0 & 1\\ -\delta_0 & -\delta_1 & -\delta_2\end{bmatrix} z. \qquad (25)$$

Here the coefficients δ_i are chosen according to the requirements to character and duration of transient of the control system (25).

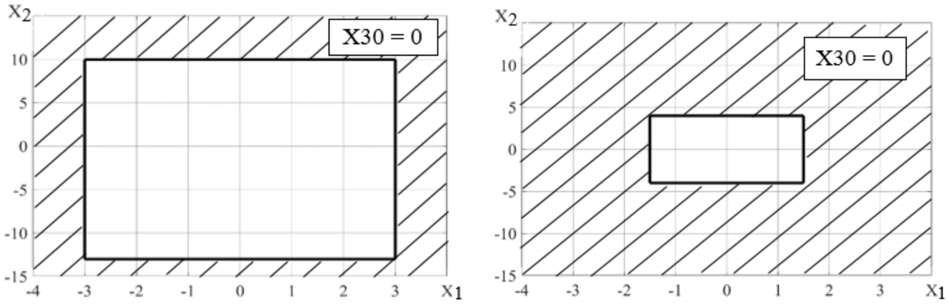
The desired stabilizing control is obtained in form

$$u_2(x)=\frac{\delta_0(2x_3-x_2)+9\delta_1\sin x_1+9\delta_2 x_2\cos x_1}{18\cos x_1}+\frac{5}{2}\sin x_1 + x_3 - \frac{x_2^2}{2}\tan x_1, \quad |x_1|<\pi/2. \qquad (26)$$

Further, we assert since the transformation $z(x)$ is reversible, i.e. a nonsingular transformation $x(z)$ exists, then the systems (14), (26) are also asymptotically stable ($\lim_{t\to\infty} x(t)=0$) if $|x_1(t,x_0)|<\pi/2$.

Modeling the systems using MATLAB with the designed control by using the two methods was shown in [13], and it was noticed that both controls make the system asymptotically stable with a duration of transient not more than 1.6 sec. Apparently from the stated above expressions the domain of equilibrium attraction of the control systems obtained using the compared methods is bounded that is caused by the controllability conditions of each method. Therefore, these systems can be operated only with bounded initial conditions.

To find the boundary of initial conditions using MATLAB, the region of initial conditions of each control system can be found in which the designed control system is still stable. This region of initial conditions of the control system is designed by the APM method shown in Fig. 1,a, and Fig. 1,b shows the same region of initial conditions of the system is designed using the feedback linearization method.



a b

Fig. 1. Cross section of region of initial conditions given by the compared methods

It is noticed that the cross section of a region of initial conditions of the system which is designed by APM method is bigger about 6 times than the same region of the designed system using feedback linearization method. Also, it can be noticed that the obtained cross section has two dimensions, where the first dimension (x-axis) describes the variable $x1$, also the variable $x2$ is described by the second dimension (y-axis) of the cross section, where this cross section was found by determining the maximum value of each variable in which the designed control system makes the system asymptotically stable, while initial condition of the third variables of the system $x_{30}=0$.

Conclusion. As a result of the study, it was found that both methods make it possible to synthesize a control system for nonlinear objects, but only if the condition of object controllability is satisfied. Nonlinear control systems obtained by both methods are asymptotically stable, but they have different regions of attraction of equilibrium. The algebraic polynomial matrix method is simpler than the feedback linearization method. This is due to the fact that the algorithm of the algebraic polynomial-matrix method has certain steps, performing which we get a nonlinear control system, spending less time and effort. At the same time, the feedback linearization method contains steps that require quite complex, but this method doesn't completely define the transformations that must be performed to obtain a mathematical model of the control system in the Brunovsky form. In this paper, it is established that the feedback linearization method leads to a nonlinear control system with a smaller range of acceptable initial conditions for state variables, in which the control system is asymptotically stable. The algebraic polynomial-matrix method gives a large range of initial conditions under which the designed control system is stable. These conclusions follow from the above cross sections of the attraction regions of both systems. In the future, it is supposed to define the region of initial conditions as a region of three-dimensional space, since the systems have an order equal to three.

БИБЛИОГРАФИЧЕСКИЙ СПИСОК

1. *Iqbal J., Ullah M., Khan S., Baizid K., Cukovic S.* Nonlinear control systems – A brief overview of historical and recent advances // Nonlinear Engineering. – 2015. – P. 301-312.
2. *Ajwad S., Iqbal J., Ullah M., Mehmood A.* A systematic review of current and emergent manipulator control approaches // Frontiers of Mechanical Engineering. – 2015. – P. 198-210.
3. *Slotine J., Li W.* Applied nonlinear control // Prentice-Hall Englewood Cliffs, NJ, 1991. – P. 1-459.
4. *Isidori A.* Lectures in Feedback Design for Multivariable Systems: Advanced Textbook in Control and Signal Processing. – London, Springer, 2016. – 414 p.
5. *Tuan H.T., Trinh H.A.* Linearized Stability Theorem for Nonlinear Delay Fractional Differential Equations // IEEE Trans. Automat. Control. – 2018. – Vol. 63, No. 9. – P. 3180-3186.
6. *Воевода А.А., Филюшов, В.В.* Линеаризация обратной связью // Сб. научных трудов Новосибирского государственного технического университета. – 2016. – № 2 (84). – С. 68-76.
7. *Лукьянов А.Г., Уткин В.И.* Методы сведения уравнений динамических систем к регулярной форме // Автоматика и телемеханика. – 1981. – Т. 4. – С. 5-13.
8. *Madeira D. de S., Adamy J.* Feedback Control of Nonlinear Systems Using Passivity Iindices. Proc // IEEE Conference on Control Applications. – Sydney, Australia, 2015. – P. 263-268.
9. *Byrnes S., Isidori A., Willems J.* Passivity, feedback equivalence, and the global stabilization of minimum phase nonlinear systems // IEEE Transactions on Automatic Control. – 1991. – Vol. 2, No. 36. – P. 1228-1240.
10. *Xia M., Rahnama A., Wang S., Antsaklis J.* Control Design Using Passivation for Stability and Performance // IEEE Transactions on control. – 2018. – Vol. 63, No. 9. – P. 2987-2993.
11. *Ascencio P., T. Astolfi T., T. Parisini T.* Backstepping PDE Design: A Convex Optimization Approach // IEEE Trans. Automat. Control. – 2018. – Vol. 63, No. 7. – P. 1943-1958.
12. *Гайдук А.Р.* Синтез нелинейных систем на основе управляемой формы Жордана // Автоматика и телемеханика. – 2006. – № 7. – С. 3-13.
13. *Gaiduk A.R.* Nonlinear Control Systems Design by Transformation Method // Mekhatronica, Avtomatizatsiya, Upravlenie, – 2018. – Vol. 19, No. 12. – P. 755-761.
14. *Гайдук А.Р.* К синтезу квазилинейных гурвицевых систем управления // Труды СПИИРАН. – 2019. – Т. 1, т. 18, № 3. – С. 678-705.
15. *Пшихопов В., Медведев М.* Позиционное управление транспортными средствами с многоконтурной адаптацией // Журнал инженерных и прикладных наук. – 2018. – №. 13. – С. 8921-8928.
16. *Капустян С.Г., Орда-Жигулина М.В., Орда-Жигулина Д.В.* Метод робастно устойчивого управления движением группы мобильных роботов с «лидером» для системного мониторинга и прогнозирования опасных процессов и обеспечения безопасности населения и береговой инфраструктуры. системы опасных процессов и обеспечения безопасности населения и береговой инфраструктуры // Наука Юга России. – 2021. – Т. 1, № 2. – С. 66-73.
17. *Gaiduk A.R., Neidorf R.A., Kudinov N.V.,* Application of the Cut-Glue Approximation in the Analytical Solution of the Nonlinear Control Synthesis Problem // Cyber-Physical Systems: Challenges of Industry 4.0. – Springer, 2020. – P. 117-132.
18. *Izadi M., Srivastava H.M.* Application of the generalized method of Bessel quasi-linearization to equations of Bratou and Lane–Emden type of arbitrary order // Fractal Fract. – 2021. – No. 5. – P. 179.
19. *Гришин А.А., Ленский А.В., Охоцимский Д.Е., Панин Д.А. Формальский, А.М.* Синтез управления для неустойчивого объекта. Перевернутый маятник // Известия РАН. Теория и системы управления. – 2002. – Т. 5. – С. 14-24.
20. *Гайдук А.Р., Капустян С.Г., Алмашаал М.Д.* Сравнение методов проектирования нелинейных систем управления // Вестник ИСПУ. – 2021. – № 4. – С. 21-24.
21. *Bela L., Lorink M.* Nonlinear control of vehicles and robots. – Springer, 2011. – 356 p.

REFERENCES

1. *Iqbal J., Ullah M., Khan S., Baizid K., Cukovic S.* Nonlinear control systems – A brief overview of historical and recent advances, *Nonlinear Engineering*, 2015, pp. 301-312.
2. *Ajwad S., Iqbal J., Ullah M., Mehmood A.* A systematic review of current and emergent manipulator control approaches, *Frontiers of Mechanical Engineering*, 2015, pp. 198-210.
3. *Slotine J., Li W.* Applied nonlinear control, *Prentice-Hall Englewood Cliffs*, NJ, 1991, pp. 1-459.

4. *Isidori A.* Lectures in Feedback Design for Multivariable Systems. *Advanced Textbook in Control and Signal Processing*. London, Springer, 2016, 414 p.
5. *Tuan H.T., Trinh H.A.* Linearized Stability Theorem for Nonlinear Delay Fractional Differential Equations, *IEEE Trans. Automat. Control*, 2018, Vol. 63, No. 9, pp. 3180-3186.
6. *Voevoda A.A., Filyushov V.Y.* Linearizatsiya obratnoy svyaz'yu [Feedback Linearization], *Sb. nauchnykh trudov Novosibirskogo gosudarstvennogo tekhnicheskogo universiteta* [Collection of scientific papers of Novosibirsk State Technical University], 2016, No. 2 (84), pp. 68-76.
7. *Luk'yanov A.G., Utkin V.I.* Metody svedeniya uravneniy dinamicheskikh sistem k regulyarnoy forme [Methods for Reducing Equations of Dynamical Systems to a Regular Form]. *Avtomatika i telemekhanika* [Automation and telemechanics], 1981, No. 4, pp. 5-13.
8. *Madeira D. de S., Adamy J.,* Feedback Control of Nonlinear Systems Using Passivity Iindices. Proc., *IEEE Conference on Control Applications*. Sydney, Australia, 2015, pp. 263-268.
9. *Byrnes S., Isidori A., Willems J*., Passivity, feedback equivalence, and the global stabilization of minimum phase nonlinear systems, *IEEE Transactions on Automatic Control*, 1991, Vol. 2, No. 36, pp. 1228-1240.
10. *Xia M., Rahnama A., Wang S., Antsaklis J*., Control Design Using Passivation for Stability and Performance, *IEEE Transactions on control,* 2018, Vol. 63, No. 9, pp. 2987-2993.
11. *Ascencio, P., T. Astolfi, T., T. Parisini, T.*, Backstepping PDE Design: A Convex Optimization Approach, *IEEE Trans. Automat. Control*, 2018, Vol. 63, No. 7, pp. 1943-1958.
12. *Gaiduk A.R.* Sintez nelineynykh system na osnove upravlyaemoy formy Zhordana [Synthesis of Nonlinear Systems Based on Controlled Jordan Form]. *Avtomatika i telemekhanika* [Automation and telemechanics], 2006, No. 7, pp. 3-13.
13. *Gaiduk A.R.* Nonlinear Control Systems Design by Transformation Method, *Mekhatronica, Avtomatizatsiya, Upravlenie*, 2018, Vol. 19, No. 12, pp. 755-761.
14. *Gaiduk A.R.* K sintezu kvazilineynykh gurvitsevykh sistem upravleniya [On Synthesis of Quasilinear Hurwitz Control Systems], *Trudy of SPIIRAN* [SPIIRAS Proceedings], 2019, Vol. 18 (3), pp. 678-705.
15. *Pshikhopov V., Medvedev M.* Position Control of Vehicles with Multi-Contour Adaptation, *Journal of Engineering and Applied Sciences*, 2018, Vol. 13, pp. 8921-8928.
16. *Kapustyan S.G., Orda-Zhigulina M.V., Orda-Zhigulina D.V.* Metod robastno ustoychivogo upravleniya dvizheniem gruppy mobil'nykh robotov c «liderom» dlya sistem monitoringa i prognozirovaniya opasnykh protsessov i obespecheniya bezopasnosti naseleniya i beregovoy infrastruktury [Method of robustness stable control of the movement of a mobile robots group with "leader" for monitoring and forecasting systems of hazardous processes and ensuring the safety of the population and coastal infrastructure], *Nauka Yuga Rossii*, [Science of the South of Russia], 2021, Vol. 17, No. 2, pp. 66-73.
17. *Gaiduk A.R., Neidorf R.A., Kudinov N.V.* Application of the Cut-Glue Approximation in the Analytical Solution of the Nonlinear Control Synthesis Problem, *Cyber-Physical Systems: Challenges of Industry 4.0*. Springer, 2020, pp. 117-132.
18. *Izadi M., Srivastava H.M.* Application of the generalized method of Bessel quasi-linearization to equations of Bratou and Lane–Emden type of arbitrary order, *Fractal Fract*., 2021, No. 5, p. 179.
19. *Grishin A.A., Lenskiy A.V., Okhotsimskiy D.E., Panin D.A. Formal'skiy, A.M.* Sintez upravleniya dlya neustoychivogo ob"ekta. Perevernutyy mayatnik [A Control Synthesis for an Unstable Object. An Inverted Pendulum]. *Izvestiya RAN. Teoriya I sistemy upravleniya* [News of the Russian Academy of Sciences. Theory and control systems], 2002, Vol. 5, pp. 14-24.
20. *Gaiduk A.R., Kapustyan S.G., Almashaal M.J.* Methods Comparison of nonlinear control systems design [Comparison of methods for designing nonlinear control systems], *Vestnik ISPU* [ISPU Bulletin], 2021, No. 4, pp.21-24.
21. *Bela L., Lorink M*. Nonlinear control of vehicles and robots. Springer, 2011, 356 p.

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